Big Apple or Bust!

A group of students from Wamsutta Middle School is planning a trip to New York City. Their student council is investigating bus companies that offer special group plans. Two advertisements appear below.

Daily trips to NYC at 6 a.m. and 10 a.m.

APPLE CHARTER
Bus Company

- $400 booking fee
- $10 per student

CITY CHARTER Bus Line

Travel to **NYC!**

- ★ *Low* **$200** booking fee
- ★ **$15** per student
- ★ Travel at 7 a.m. or NOON daily !

1. Which bus company plan do you think is the better deal? Explain why you think so.

2. Complete the table below.

Cost Comparison

Number of Students	Apple Charter	City Charter
5		
15		
25		
35		
45		

3. Now use the table to describe which bus company you think is the better deal, and compare it to your answer in question 1.

4. On the grid in the next column, use two different colors to graph the cost of 5, 15, 25, 35, and 45 students. Use one color to represent the Apple Charter Bus Company and another color for the City Charter Bus Line to NY City.

Bus Company Plans
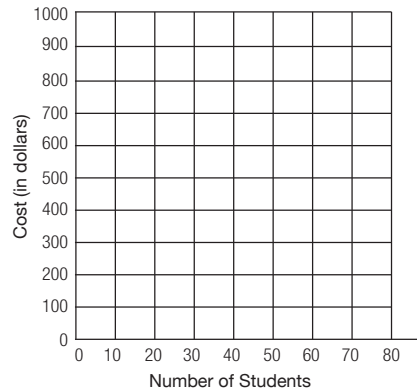
5. Describe in words how to find the cost of using Apple Charter Bus Company with any number of students:

   The cost will equal _____ plus _____

   _____

6. Use variables and numbers to write an equation that describes the cost of using Apple Charter Bus Company with any number of students. Let C represent the cost and s the number of students:

   $C = $ _____

7. Use the same procedure that you followed in questions 5 and 6 to write an equation that describes the cost of using City Charter Bus Line with any number of students. Let C represent the cost and s the number of students.

   $C = $ _____

8. Use the equations that you wrote to find the cost for transporting 23 students with each bus company.

9. Use the equations that you wrote to find the cost of 68 students with each bus company.

10. What do the answers to questions 8 and 9 tell you about which bus company has the better deal?

We can use any of the representations to find the number of students that will cost any fixed amount of money.

11. Use your equation from question 7 to find the number of students who can attend for $830 with the City Charter Bus Line.

12. Explain how you can use either the table or your graph to confirm your answer to question 11.

13. Use any representation to find the number of students who can attend for $830 with the Apple Charter Bus Company. Explain your choice.

Using a graph, table, or equation also allows us to find when the cost would be the same for the same number of students with both bus companies.

14. Use your graph representation to find the number of students for which the cost would be the same with both bus companies. What do you see on the graph that indicates this?

15. Use the equations that you wrote for questions 6 and 7 to confirm that the cost would be the same for that number of students.

The two equations that you wrote describing the cost (*C*), for any number of students (*s*), is considered a system of equations. The solution to a system of equations is an ordered pair (number of students, cost) that satisfies both equations in the system.

16. Use your graph, tables, or equations to find the interval of the number of students that would make the Apple Charter Bus Company the better deal. Explain which representation you used and how you determined your answer.

17. Find the interval of the number of students that would make the City Charter Bus Line the better deal. Which representation did you use? How did you determine your answer?

Accommodations and Sightseeing Plans

After the student council chose a bus company, it was time to investigate package prices for hotel accommodations and museum tickets. Assume that for both packages below, each night's hotel cost is the same and each museum ticket cost is the same.

PACKAGE A 1 night's hotel accommodation and 2 museum tickets for **$170** per person

PACKAGE B 1 night's hotel accommodation and 3 museum tickets for **$200** per person

18. Notice that we already know the total cost per person of each package. What accounts for the difference in the cost of the two packages?

19. Find the cost for 1 museum ticket for each package.

20. Find the cost for 1 night's hotel accommodation for each package.

21. Suppose you receive a brochure in the mail describing 2 new packages. Package C is for 2 nights and 2 museum tickets. Assume that the cost for each hotel accommodation and each museum ticket is the same. Find the total cost per person for Package C.

Big Apple or Bust—*continued*

22. Package D includes 2 nights and 4 museum tickets. Find the total cost per person for Package D.

23. Write an equation that represents the cost in package A. Let h represent the cost of nightly accommodations and m represent the cost of museum tickets.

    $170 = \underline{\hspace{6cm}}$

24. Write an equation to represent the costs in Package B. Let h represent the cost of nightly accommodations and m represent the cost of museum tickets.

    $200 = \underline{\hspace{6cm}}$

Other Ways to Solve Systems

If two equations are true, then the sum of the equations must be true:

    If $2 + 3 = 5$

    and $+ (2 + 4 = 6)$

    then $4 + 7 = 11.$

Likewise, the difference between the two equations must be true:

    If $2 + 3 = 5$

    and $- (2 + 4 = 6)$

    then $0 + (-1) = -1.$

Now let's use the two equations that you wrote in questions 23 and 24 to find another way to solve the system. Remember that the solution to the system is the ordered pair (h, m), or (hotel cost, museum cost), that makes both equations true.

25. Align the equations you wrote in questions 24 and 25. Then subtract one equation from the other by subtracting like terms. Find the cost of each museum ticket and check it with your previous answer.

26. Use the information in either equation to find the cost for each night's hotel stay. Check it with your previous answer.

Sometimes equations are not quite so simple, and adding or subtracting like terms will not immediately allow us to see the solution. If we multiply any equation by a constant number, the solution remains true. Notice that if we multiply each number in the equation $3 + 5 = 8$ by 2, the new equation, $6 + 10 = 16$, is also true.

In this way, we can change the value of a coefficient or constant of either equation so that the transformed equation can help us determine the solution.

(A *coefficient* is the number that is multiplied by the variable; for example, in $2h$, 2 is the coefficient.)

Suppose another company offers the following package prices for a hotel stay and sightseeing tour tickets.

Package X	3 night's hotel accommodation and 2 sightseeing tickets for **$415** per person
Package Y	4 night's hotel accommodation and 4 sightseeing tickets for **$620** per person

27. Write an equation for Package X using h to represent the cost of nightly hotel accommodations and t to represent the cost of the sightseeing tour tickets.

28. Write an equation for Package Y using h to represent the cost of nightly hotel accommodations and t to represent the cost of the sightseeing tour tickets.

29. Solve the system of equations to find the cost of the nightly hotel accommodations and the cost of one sightseeing tour ticket.

Can You …

- cite other real-life examples of systems of equations?
- solve a system of 3 equations?
- solve the following system of equations?

    $$3x = 161 - 5y$$
    $$2x = 105 - 3y$$

Mathematical Content

Tables, graphs, equations, systems of equations

Resource

Opening the Gate Activities. Florida Department of Education, 1992–1993.
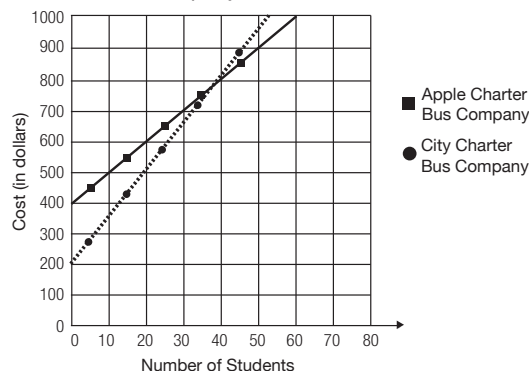
Answers

1. Answers may vary. For example, the Apple Charter Bus Company appears to be less expensive for a large number of students.

2.

Cost Comparison

Number of Students	Apple Charter	City Charter
5	$450	$275
15	$550	$425
25	$650	$575
35	$750	$725
45	$850	$875

3. Answers may vary. For example, the Apple Charter Bus Company still appears to be the better deal.

4.

Bus Company Plans

5. Cost = $400 plus $10 times the number of students.

6. $C = 400 + 10s$ **7.** $C = 200 + 15s$

8. On the Apple Charter Bus Company, 23 students cost $630. On the City Charter Bus Line, 23 students cost $545.

9. On the Apple Charter Bus Company, 68 students cost $1080. On the City Charter Bus Line, 68 students cost $1220.

10. Answers may vary. Sample answer: Sometimes the Apple Bus Company is the better deal; other times the City Charter Bus Line is the better deal.

11. For $830, 42 students can travel on the City Charter Bus Line.

12. A sample answer might state, "If you trace a horizontal line across from where $C = 830$ to the points representing City Charter Bus Line, the vertical line from that point crosses the s axis at 42 students.

13. With the Apple Charter, 43 students can travel. A sample explanation might state, "I chose the equation because I can calculate the exact number of students," or the answer may show the work to solve $400 + 10s = 830$ or be represented by a table or graph.

14. When 40 students go on the trip, the cost is equal on both bus lines. The point where the lines intersect represents the same number of students and the same cost on both lines.

15. $200 + 15s = 400 + 10s$

$$5s = 200$$
$$s = 40$$

16. When $s > 40$, the Apple Charter is the better deal. Explanations can be from a table, graph, or equation.

17. When $s < 40$, the City Charter is the better deal. Explanations can be from a table, graph, or equation.

18. The difference is the number of museum tickets.

19. $200 – $170 = $30

20. $110   [Package A $170 – (2 • 30); Package B $200 – (3 • 30)]

21. The total cost per person for Package C is $280.

22. The total cost per person for Package D is $340.

23. $170 = 1h + 2m$ **24.** $200 = 1h + 3m$

25.
$$170 = 1h + 2m$$
$$-(200 = 1h + 3m)$$
$$-30 = -1m$$
$$30 = m$$

26.
$$170 = 1h + 2(30)$$
$$170 = 1h + 60$$
$$110 = 1h$$

27. $3h + 2t = $415   **28.** $4h + 4t = $620

29.

	(X)	$3h + 2t = 415$ (multiply by 2)
	(Y)	$4h + 4t = 620$
therefore	(2X)	$6h + 4t = 830$
	(Y) –	$(4h + 4t = 620)$

$$2h = 210$$
$$h = 105$$

So, the cost of one night in the hotel is $105.

If $3h + 2t = 415$
$$3(105) + 2t = 415$$
$$315 + 2t = 415$$
$$2t = 100$$
$$t = 50$$

Therefore, each tour ticket is $50.

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